Lecture 16: The Physics of Control (Delayed Neutrons and the Inhour Equation)

CBE 30235: Introduction to Nuclear Engineering — D. T. Leighton

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Reading Assignment

Lamarsh & Baratta (4th Edition):

- **Section 7.2** Reactor Kinetics (Prompt/Delayed Neutrons, Inhour Equation, Prompt Critical).

Recap: The Problem with Prompt Neutrons

In Lecture 15, we saw that if a reactor relied solely on prompt neutrons, the time scale of the reaction (the Reactor Period T) would be on the order of milliseconds.

$$T \approx \frac{l}{\rho} \quad where \quad \rho = \frac{k-1}{k}$$

If $l = 10^{-4}$ s and $\rho = 0.001$, the power rises by a factor of 22,000 in one second. This is uncontrollable. Today, we mathematically prove how **Delayed Neutrons** save us.

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1 The Point Kinetics Equations

To account for delayed neutrons, we must modify our neutron balance equation. We split the neutron source into two terms:

1. **Prompt Neutrons:** $(1-\beta)$ fraction of fission neutrons appear instantly.
2. **Delayed Neutrons:** β fraction of fission neutrons appear later, coming from the decay of Precursors (C).

We now need two coupled differential equations: one for the Neutrons (n) and one for the Precursors (C).

1.1 1. The Neutron Equation

$$\frac{dn}{dt} = \underbrace{\frac{k(1-\beta)n}{l}}_{\text{Prompt Generation}} + \underbrace{\lambda C}_{\text{Delayed Source}} - \underbrace{\frac{n}{l}}_{\text{Loss}}$$

Here, λ is the decay constant of the precursor (e.g., ^{87}Br).

1.2 2. The Precursor Equation

The precursors are created by fission and lost by radioactive decay.

$$\frac{dC}{dt} = \underbrace{\frac{k\beta n}{l}}_{\text{Production}} - \underbrace{\lambda C}_{\text{Decay}}$$

2 The Solution: The Inhour Equation

We look for solutions of the form $n(t) = n_0 e^{t/T}$ (exponential growth with period T). Substituting this ansatz into the coupled system involves some algebra (omitted here for brevity, see Lamarsh Ch. 7), but the result is the fundamental equation of reactor control:

$$\rho = \frac{l}{Tk_{eff}} + \frac{\beta_{eff}}{1 + \lambda T} \tag{1}$$

The Inhour Equation (Simplified One-Group Form)

This equation relates the **Reactivity** (ρ, what the operator changes with control rods) to the **Reactor Period** (T, how fast the power changes). Recall The reactor period T is the time required for power to change by a factor of e.

Note: In a real reactor, there are 6 distinct groups of precursors with different decay constants λ_i. The full equation is a sum:

$$\rho = \frac{l}{T} + \sum_{i=1}^{6} \frac{\beta_i}{1 + \lambda_i T}$$

3 Analyzing the Result

Let's analyze the One-Group equation to understand the three distinct regimes of reactor operation.

3.1 Regime 1: Sub-Prompt Critical (Normal Operation)

Suppose we insert a very small amount of reactivity: $\rho \ll \beta$. Assume $\rho = 0.001$ (0.1%). For U-235, $\beta \approx 0.0065$ (0.65%). Since l is tiny (10^{-4}), the first term l/T is negligible for large T. The equation simplifies to:

$$\rho \approx \frac{\beta}{1 + \lambda T}$$

Solving for Period T:

$$T \approx \frac{\beta - \rho}{\lambda \rho}$$

Using $\lambda \approx 0.1\ \text{s}^{-1}$ (average precursor decay constant):

$$T \approx \frac{0.0065 - 0.001}{(0.1)(0.001)} = \frac{0.0055}{0.0001} = \mathbf{55}\ \text{seconds}$$

Result: Instead of 0.1 seconds (Prompt), the period is 55 seconds. This is slow! A human operator can easily watch a gauge and adjust a rod in 55 seconds.

3.2 Regime 2: The Threshold (Prompt Critical)

What happens if we add enough reactivity such that $\rho = \beta$?

$$\beta \approx \frac{l}{T} + \frac{\beta}{1 + \lambda T}$$

The math blows up. T must become very small for this equality to hold. Physically, this means the reactor is critical on prompt neutrons *alone*. The delayed neutrons are no longer needed to sustain the chain reaction.

3.3 Regime 3: Super-Prompt Critical (The Danger Zone)

If $\rho > \beta$, the reactor is supercritical on prompt neutrons.

$$\rho \approx \frac{l}{T}$$

We return to the "Bomb" kinetics of Lecture 15. The period drops to milliseconds with only a tiny increase in the reactivity.

$$T \approx \frac{10^{-4}}{0.001} = 0.1 \text{ s}$$

4 Units of Reactivity: The Dollar

Because the fraction β is the most important safety limit in a reactor, we use it as a unit of measurement.

- **1 Dollar ($1.00)** = A reactivity of β.
- **1 Cent (1¢)** = $0.01 \times \beta$.

For U-235 ($\beta = 0.0065$):

- $\$1.00 = 0.0065 \Delta k/k$
- $\$0.50 = 0.00325 \Delta k/k$ (Safe)
- $\$1.10 = 0.00715 \Delta k/k$ (Prompt Critical - Dangerous)

"The reactor is rising on a 50-cent period" is standard control room terminology.

5 Physical Insight: Why does it work?

Think of the delayed neutrons as a "storage buffer."

- When you increase reactivity, the prompt neutrons rise instantly.
- But the delayed neutrons (0.65% of the population) are "stuck" in the precursors (Bromine/Iodine atoms). They haven't been born yet.
- The total population cannot rise to the new equilibrium level until those precursors decay and release their neutrons.
- This "drag" holds the reactor back, forcing it to wait for the precursors (approx. 10 to 80 seconds).

6 Summary

1. **Point Kinetics:** We couple the neutron population to the precursor concentration.
2. **The Inhour Equation:** Relates Reactivity (ρ) to Period (T).
3. **Safety Limit:** We must keep $\rho < \beta$ (less than $1.00).
4. **Normal Operation:** As long as $\rho < \beta$, the reactor period is determined by the slow decay of precursors (λ), making the reactor controllable by humans.